

# The family of photon subtracted thermal states: description, preparation and reconstruction

Yu. I. Bogdanov,<sup>1,2,3</sup> K. G. Katamadze,<sup>1,4,\*</sup> G. V. Avosopyants,<sup>1,3,4</sup>  
L. V. Belinsky,<sup>1,3</sup> N. A. Bogdanova,<sup>1,3</sup> A. A. Kalinkin,<sup>4</sup> and S. P. Kulik<sup>4</sup>

<sup>1</sup>*Institute of Physics and Technology, Russian Academy of Sciences, 117218, Moscow, Russia*

<sup>2</sup>*National Research Nuclear University "MEPHI", 115409, Moscow, Russia*

<sup>3</sup>*National Research University of Electronic Technology MIET, 124498, Moscow, Russia*

<sup>4</sup>*M. V. Lomonosov Moscow State University, 119991, Moscow, Russia*

We present a family of optical quantum states generated by subtraction of photons from the thermal state. The experimental realization of their preparation, measurement, and quantum state reconstruction is demonstrated. The proposed technique allows generation of up to 10-photon subtracted thermal states with the fidelity higher than 99%. Combined with homodyne detection it can also be used for precise measurement of high-order autocorrelation functions.

PACS numbers: 03.65.Wj, 03.67.a, 42.50.-Dv

Keywords: quantum optics; homodyne detection; quadrature measurement; quantum state tomography; Wigner function; thermal state; photon subtraction; compound Poisson distribution

## INTRODUCTION

Preparation and measurement of various quantum states of light are the key stones of quantum optics. So far only several classes of quantum states were available for experimental research. Among them there are displaced and squeezed states, the first few Fock states, Schrodinger cat states and several more. One of them, the thermal state, plays a very special role. On the one hand, it is an easy-to-prepare state, but on the other, it supports classical correlations and can be used as a testing area for some effects based on classical or quantum correlations.

The first pioneer experiment in quantum optics is considered to be the work by Hanbury Brown and Twiss [1], who investigated correlations in thermal light by means of a beam splitter and a pair of detectors, outputs of which are analyzed with a coincidence circuit. Since then thermal states have been used in many applications including ghost imaging [2–4], quantum illumination [5], and “thermal laser” [6]. Schmidt-like correlations were also observed in thermal states [7]. Recently thermal states have been used in the classical analog of quantum teleportation [8]. In the present paper we demonstrate a family of thermal states modified by multiphoton subtraction.

Photon adding and subtraction is of the great interest in quantum optics, because it provides a tool for direct tests of basic commutation relations [9], enables Schrodinger cat and other exotic quantum state preparation [10]. Also it can be used for probabilistic linear no-noise amplification [11]. One- and two-photon subtracted thermal states were demonstrated for the first time in [12] but in the present work we provide a comprehensive description of multiphoton subtracted thermal states, based on a general approach, suitable for any photon number distribution. We demonstrate the tech-

nique of high-fidelity preparation and reconstruction of up to 10-photon subtracted thermal states. The technique discussed in the paper can also be used in some metrological applications [13, 14].

## PHOTON SUBTRACTED STATES

Photon number distribution  $P(n)$  is a key characteristic of any quantum state of light. Any particular distribution corresponds to its generating function  $G(z)$ , which can be defined by equation:

$$G(z) = \sum_n P(n) z^n, P(n) = \frac{G^{(n)}(0)}{n!}, \quad (1)$$

where  $G^{(n)}$  is an  $n$ -th order derivative. Properties of the annihilation operator and renormalization conditions lead us to the simple description of photon subtraction:

$$G_1(z) = \frac{G^{(1)}(z)}{\mu}, \quad (2)$$

where  $G_1(z)$  is the generating function, which corresponds to the photon subtracted state and  $\mu$  is a mean photon number of the initial state. Applying (2)  $k$  times, one can find the generating function for the  $k$ -photon subtracted state:

$$G_k(z) = \frac{G^{(k)}(z)}{\mu \mu_1 \cdots \mu_{k-1}}, \quad (3)$$

where  $\mu_k$  is a mean photon number of  $k$ -photon subtracted state.

Equations (2) and (3) can be used for calculation of the distribution  $P(n)$  (1) as well as for the  $m$ -th order correlation function calculation:

$$g^{(m)} = \frac{G^{(m)}(1)}{\mu^m} = \frac{\mu_1 \mu_2 \cdots \mu_{m-1}}{\mu^{m-1}}, m = 2, 3, \dots \quad (4)$$

Lets consider several examples.

### 1. Fock state

The photon number distribution of the Fock state  $|m\rangle$  is  $P(n) = \delta_{m,n}$  and its generating function  $G(z) = z^m$ . After photon subtraction (2) it transforms to  $G_1(z) = z^{m-1}$ , which corresponds to the state  $|m-1\rangle$ .

### 2. Coherent state

A coherent state can be written in the Fock basis as  $|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$ , so its photon number has a Poisson distribution  $P(n) = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!}$  with the mean photon number  $\mu = |\alpha|^2$  so the generating function turns  $G(z) = e^{\mu(z-1)}$ . Applying photon subtraction (2) one can verify that  $G_1(z) = G(z)$ , which means that coherent state doesn't change under photon subtraction.

### 3. Squeezed vacuum

The photon number distribution of the squeezed vacuum state  $\hat{S}(\xi)|0\rangle$  is [15]

$$P(2n) = \frac{1}{\cosh(|\xi|)} \frac{(2n)!}{(n!)^2} \left( \frac{1}{2} \tanh(|\xi|) \right)^{2n}, \quad (5)$$

$$P(2n+1) = 0, n = 0, 1, \dots$$

Its generating function equals

$$G(z) = \frac{1}{\cosh(|\xi|) \sqrt{1 - z^2 \tanh^2(|\xi|)}}, \quad (6)$$

and its mean photon number is  $\mu = G^{(1)}(1) = \sinh^2(|\xi|)$ .

Using our approach, we can, for example, calculate a high-order correlation function of squeezed vacuum:

$$g^{(n)} = \frac{n!}{2^n} \sum_{k=0}^{\lfloor n/2 \rfloor} \frac{(2n-2k)!}{k!(n-k)!(n-2k)!} \left( \frac{1}{\sinh^2(|\xi|)} \right)^k, \quad (7)$$

where  $\lfloor \dots \rfloor$  is the floor function.

### 4. Thermal state

The density matrix of a thermal state has a well-known diagonal form:

$$\hat{\rho} = \sum_{n=0}^{\infty} P(n) |n\rangle \langle n|, \quad (8)$$

where  $P(n) = \frac{\mu^n}{(1+\mu)^{n+1}}$  is a Bose-Einstein distribution. This distribution is a particular case of compound Poisson distribution

$$P(n) = \frac{\Gamma(a+n)}{\Gamma(a)} \frac{\mu^n}{a^n n!} \frac{1}{(1+\mu/a)^{n+a}}. \quad (9)$$

This distribution has two parameters: the mean photon number  $\mu$  and coherence parameter  $a$ . At  $a = 1$  equation (9) turns into the Bose-Einstein distribution, and at  $a \rightarrow \infty$  (9) turns into the ordinary Poisson distribution. This distribution describes a multimode thermal state, where  $a$  is the number of modes [16].

It can be shown, that the same distribution applies also to the single-mode multiphoton-subtracted thermal state [17].

Its generating function equals:

$$G(z) = (1 + (1-z)\mu/a)^{-a}. \quad (10)$$

Using (2) one can show that photon subtraction conserves the type of the distribution, (9) but changes the values of parameters  $a$  and  $\mu$  as follows:  $a_1 = a + 1$ ,  $\mu_1 = \mu \frac{a+1}{a}$ . Using these iterative relations we can see that a thermal state with the initial parameters  $\mu_0$  and  $a_0 = 1$  after subtraction of  $k$  photons transforms into the state (8), (9) with parameters

$$a_k = k + 1, \mu_k = \mu_0(k + 1). \quad (11)$$

It is quite counterintuitive that the mean photon number increases after photon subtraction. It can be explained as follows. Probabilistic photon subtraction can be realized by means of a low-reflective beam splitter combined with a single-photon detector in the reflection channel, which clicks if the photon annihilation takes place [10]. As the reflection of the beam splitter is very weak, most of the time there are no detector clicks. However, when a photon is detected it results in the following: 1. there is one less photon after the beam splitter than before; 2. the number of photons before the beam splitter was greater (on the average) than the mean. In our case the second factor is much greater than the first one. Let us mention, that for coherent states with Poisson photon distributions these two factors compensate each other, so the photon subtraction doesn't change the mean photon number.

This peculiar behavior can be effectively used as probabilistic amplification due to photon subtraction, which enables higher phase sensitivity in thermal field interferometry [13, 14].

In contrast, ordinary losses only decrease  $\mu$  and conserve  $a$ .

Using (4), we can show that the correlation function of a  $k$ -photon subtracted thermal state equals

$$g^2 = 1 + \frac{1}{a} = 1 + \frac{1}{k+1}. \quad (12)$$

This equation is similar to the correlation function for multi-mode thermal state [16].

Photon number distributions for several photon-subtracted thermal states as well as their Wigner functions are shown in Fig. 1. Following the procedure of

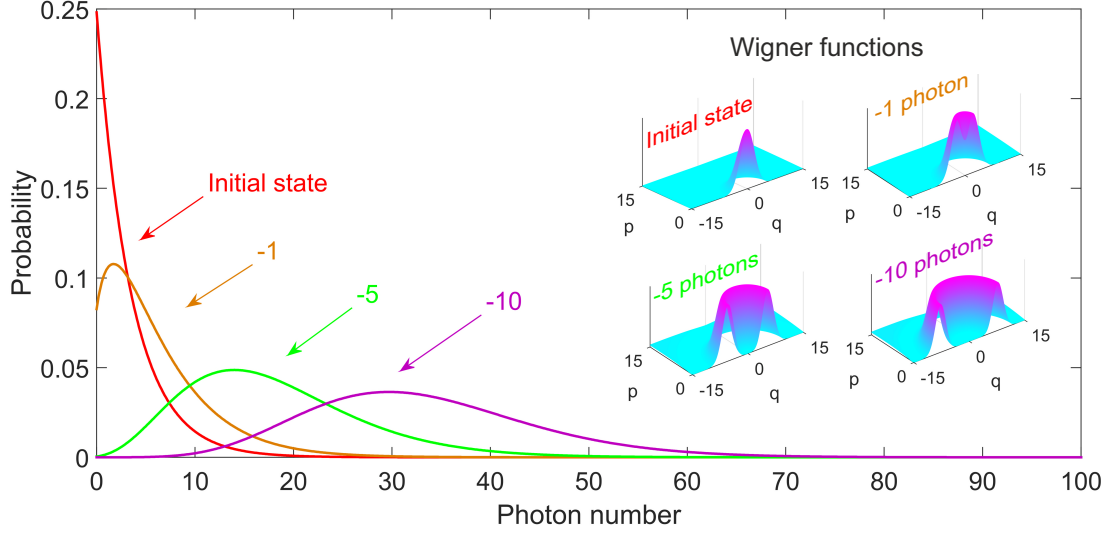


Figure 1. Photon number distributions and Wigner functions for initial thermal state and  $k$ -photon subtracted thermal states with  $k = 1, 5, 10$ .

photon subtraction, the initial Gaussian function transforms to a ring-shaped function, whose radius is approximately proportional to  $\sqrt{\mu_k}$ .

The example of a quadrature distribution  $P(q)$  for an 8-photon subtracted thermal state is shown in Fig. 2. It can be shown that its variance  $\sigma^2$  and the excess kurtosis  $K \equiv (\overline{(q - \bar{q})^4})/\sigma^4 - 3$  relates to photon distribution parameters  $a$  and  $\mu$  as

$$\sigma^2 = \mu + \frac{1}{2}, K = -6 \left( \frac{\mu}{2\mu + 1} \right)^2 \frac{a - 1}{a}. \quad (13)$$

These relations can be used for quick estimation of  $a$  and  $\mu$  from homodyne measurements.

## EXPERIMENT

The sketch of the experimental setup is shown in Fig. 3. The initial quasi-thermal state is prepared by passing radiation of a cw He-Ne laser with the wavelength of 633 nm through the rotating ground glass disk [18, 19]. The corresponding coherence time of  $\tau_{coh} = 60 \mu s$  approximately equals the time it takes for a point of the disc to cross the laser beam. Photon subtraction is realized by a beam splitter with reflectivity  $r = 1\%$  combined with an single photon detector, based on an avalanche photodiode (APD) placed in the reflection channel. Finally, the quadrature distribution of the obtained photon subtracted thermal state is measured with the homodyne technique [20]. The registration averaging time equals  $\tau_a = 15 \mu s$ . Although our setup is very similar to the one in [12], we use the cw mode instead of the pulsed one to satisfy the condition  $\tau_d \ll \tau_a < \tau_{coh}$ , where  $\tau_d = 20 ns$

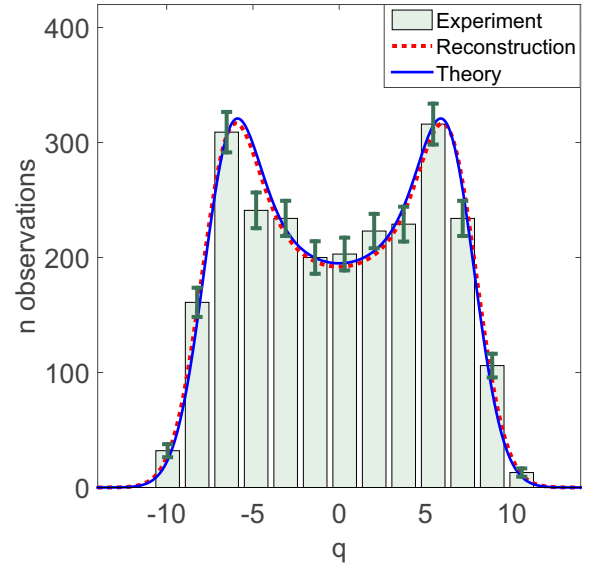


Figure 2. Quadrature distribution  $P(q)$  for the 8-photon subtracted thermal state. Experimental data are plotted as a histogram with statistical errors, the MLE fit is plotted as a red dashed line and theoretical distribution as a blue solid line.

is the APD dead time. This operation mode enables to register several photo-counts during the averaging time, which effectively results in multiple photon subtraction. Using this technique we were able to prepare up to ten-photon subtracted thermal states. Alternatively, a photon number resolving detector could be used for optical fields with a small correlation time.

The measured conditional quadrature distributions

were used to reconstruct the prepared quantum states of light.

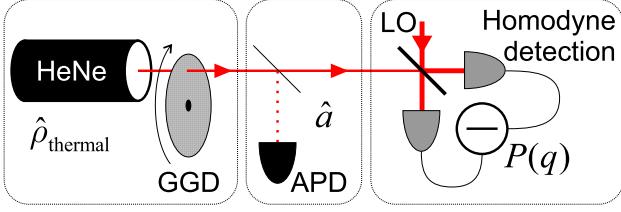


Figure 3. Experimental setup. Thermal state  $\rho_{\text{thermal}}$  is prepared from a HeNe laser radiation by randomizing its phase in a rotating ground glass disk (GGD) [18, 19]. Photon subtraction  $\hat{a}$  is realized with a low-refractive beam splitter combined with a single-photon APD detector. The quadrature distribution of prepared state is measured with the homodyne detection technique [20].

## RECONSTRUCTION

An easy way to reconstruct the quantum state (8), (9) from experimental quadrature data is based on the relations (13). However, more accurate results can be obtained with the maximum likelihood estimation (MLE). Typically, the MLE is used to reconstruct the density matrix of the state  $\hat{\rho} = \sum_{n,m=0}^N \rho_{n,m} |n\rangle \langle m|$ , where  $N$  is a limit of maximum photon number [21]. This model is quite general, but not optimal, because it has a too large number of estimated parameters; the corresponding problem is ill-conditioned and requires a lot of computing power. Therefore, it gives rather low precision of estimates. For a considerable set of experimentally available quantum states of light the model, based on the basis of displaced squeezed Fock states and root approach can be used for significant decrease the number of estimated parameters [22]. However, for the purposes of this paper a more simple model is sufficient that is based on the compound Poisson photon number distribution (8), (9) and has only two parameters  $a$  and  $\mu$ . This model was validated using the usual  $\chi^2$  - test. The significance level was higher than 0.01 for all of the prepared and measured states. In Fig. 2 one can see that the dashed red line, obtained by MLE, is a good fit for the experimental quadrature data, plotted as a histogram, and lays close to the solid blue line, which corresponds to the state (8), (9) with theoretically predicted values of  $a$  and  $\mu$ .

## RESULTS

Eleven different quantum states were prepared, measured and reconstructed: the initial thermal state with the mean photon number  $\mu = 3$ , and  $k$ -photon subtracted

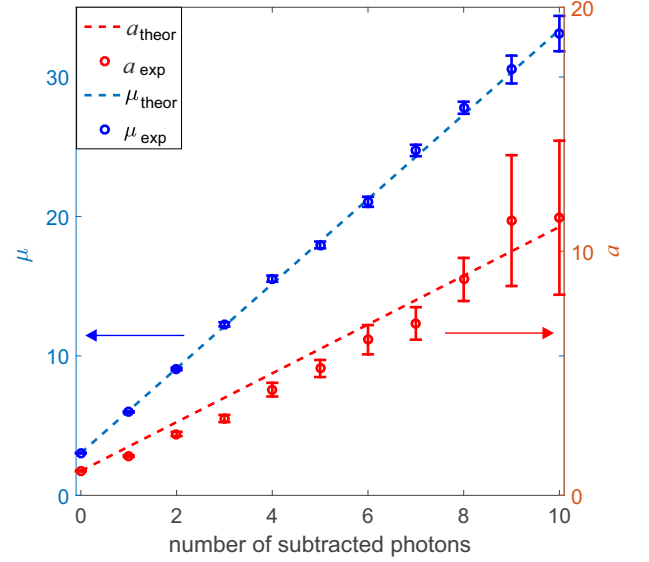


Figure 4. Dependency of the mean photon number  $\mu$  and coherence parameter  $a$  on the number of subtracted photons. Dots corresponds to experimental values and dashed lines to theoretical predictions (11).

thermal states, where  $k = 1, \dots, 10$ . The estimated values of  $a$  and  $\mu$  are plotted in Fig. 4. Dashed lines correspond to the predicted values of the parameters (11). As follows from the figure, experimental results are in the good agreement with theoretical predictions.

Error bars of the estimated parameters were calculated using the Fisher information matrix. Large uncertainties for  $k = 9, 10$  are due to the small volume of the sampled data (just 500 and 358 points).

We estimate the agreement between theoretical and experimental density matrices by calculating the fidelity:

$$F(\hat{\rho}_{th}, \hat{\rho}_{exp}) = \left( \text{Tr} \left( \sqrt{\sqrt{\hat{\rho}_{th}} \hat{\rho}_{exp} \sqrt{\hat{\rho}_{th}}} \right) \right)^2 \quad (14)$$

For all the measured states the fidelity is higher than 99%. The calculated values of infidelity ( $1 - F$ ) are presented in Table I.

We should mention that quadrature data always suffer from setup imperfections, such as the limited APD efficiency and dark counts. Nonetheless, the estimated distributions are in excellent agreement with the data, while the fidelity with the ideal states is relatively high.

## CONCLUSION

A family of photon-subtracted thermal states has been presented based on the compound Poisson photon number distribution. Up to ten-photon subtracted states have been experimentally realized with a single APD by means of a long coherence time of the initial thermal

Table I. State reconstruction parameters

Number of subtracted photons	Sample size	Infidelity ( $1 - F$ )
0	50 000	$1.1 \cdot 10^{-4}$
1	25 000	$3.4 \cdot 10^{-3}$
2	12 500	$2.2 \cdot 10^{-3}$
3	7 500	$4.3 \cdot 10^{-3}$
4	4 500	$2.0 \cdot 10^{-3}$
5	4 500	$1.7 \cdot 10^{-3}$
6	2 500	$7.2 \cdot 10^{-4}$
7	2 500	$1.7 \cdot 10^{-3}$
8	2 500	$5.5 \cdot 10^{-4}$
9	500	$1.0 \cdot 10^{-3}$
10	358	$2.1 \cdot 10^{-4}$

state. All the prepared states were statistically reconstructed with  $> 99\%$  fidelity.

This work was supported by Russian Foundation of Basic Research (project no: 14-02-00749 A) and by the Grant of President of Russian Federation no: MK-5860.2016.2.

This work was also supported by the Program of the Russian Academy of Sciences in fundamental research.

---

\* k.g.katamadze@gmail.com

- [1] R. H. Brown and R. Q. Twiss, *Nature* **177**, 27 (1956).
- [2] A. Gatti, E. Brambilla, M. Bache, and L. A. Lugiato, *Phys. Rev. Lett.* **93**, 093602 (2004).
- [3] F. Ferri, D. Magatti, A. Gatti, M. Bache, E. Brambilla, and L. A. Lugiato, *Phys. Rev. Lett.* **94**, 183602 (2005).
- [4] A. Valencia, G. Scarcelli, M. D'Angelo, and Y. Shih, *Phys. Rev. Lett.* **94**, 063601 (2005).
- [5] S. Lloyd, *Science* **321**, 1463 (2008).
- [6] M. V. Chekhova, S. P. Kulik, A. N. Penin, and P. A. Prudkovskii, *Phys. Rev. A* **54**, R4645 (1996).
- [7] I. B. Bobrov, S. S. Straupe, E. V. Kovlakov, and S. P. Kulik, *New J. Phys.* **15**, 73016 (2013).
- [8] D. Guzman-Silva et al., *Laser Photonics Rev.* **321**, 317 (2016).
- [9] V. Parigi, A. Zavatta, M. Kim, and M. Bellini, *Science* **317**, 1890 (2007).
- [10] J. Wenger, R. Tualle-Brouiri, and P. Grangier, *Phys. Rev. Lett.* **92**, 153601 (2004).
- [11] G. Y. Xiang, T. C. Ralph, A. P. Lund, N. Walk, and G. J. Pryde, *Nat. Photonics* **4**, 316 (2010).
- [12] A. Zavatta, V. Parigi, M. S. Kim, and M. Bellini, *New J. Phys.* **10**, 123006 (2008).
- [13] C. G. Parazzoli and Barbara A. Capron, in *Conf. Lasers Electro-Optics (OSA, Washington, D.C., 2016)*, p. FTu3C.4.
- [14] S. M. H. Rafsanjani et. al., arXiv:1605.05424, (2016)
- [15] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, 2001).
- [16] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, 1995).
- [17] Yu. I. Bogdanov, N. A. Bogdanova, K. G. Katamadze, G. V. Avosopyants, and V. F. Lukichev, *Avtometriya*, **52**, 5, pp. 71-83, (2016) [Optoelectronics, Instrumentation and Data Processing, **52**, 5, (in press)].
- [18] W. Martienssen, *Am. J. Phys.* **32**, 919 (1964).
- [19] F. T. Arecchi, *Phys. Rev. Lett.* **15**, 912 (1965).
- [20] U. Leonhardt and H. Paul, *Prog. Quantum Electron.* **19**, 89 (1995).
- [21] A. I. Lvovsky, *J. Opt. B Quantum Semiclassical Opt.* **6**, S556 (2004).
- [22] Yu. I. Bogdanov, G. V. Avosopyants, L. V. Belinskii, K. G. Katamadze, S. P. Kulik, and V. F. Lukichev, *Journal of Experimental and Theoretical Physics*, **123**, 2, pp. 212-218, (2016).